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ABSTRACT

This publication contains the conference proceedings of the New Zealand Council for Educational Research's (NZCER) mathematics seminar in 1998. Practical insights are offered into topics such as Maori students' participation and performance in mathematics, a conceptual development for multiplication using thinking strategies, practical processes for developing number sense, and using the Assessment Resource Banks in mathematics to help teaching and learning. This publication contains these major papers: (1) "Mathematics Education in New Zealand: Have We Failed Our Maori Students?" (Megan Clark); (2) "A Conceptual Development for Multiplication: Thinking Strategies in Place of Table Memorisation" (George Booker); (3) "Promoting Number Sense Rather Than Simply Providing Number Skills" (George Booker); and (4) "Assessment Resource Banks in Mathematics: How Can Testing Help Teaching and Learning?" (W. Alex Neill). (Each paper contains references.) (ASK)

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MATHEMATICS TEACHING: WHERE ARE WE AT?

SEMINAR PROCEEDINGS
MAY 1998

Compiled by Bev Webber

New Zealand Council for Educational Research
Wellington
1999

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INTRODUCTION

Are we failing our Maori students? Can we replace table memorisation with thinking strategies? How can testing help teaching and learning? How can we develop number sense rather than number skills for our students?

These are some of the key questions which were addressed by the speakers at NZCER's mathematics seminar in 1998. Dr Megan Clark, Victoria University; Professor George Booker, Griffith University; and Alex Neill, New Zealand Council for Educational Research; offered practical insights into these topics: Maori students' participation and performance in mathematics, a conceptual development for multiplication using thinking strategies, practical processes for developing number sense, and using the Assessment Resource Banks in mathematics to help teaching and learning.

This publication contains these major papers. It is hoped they will contribute to improved teaching and learning for many teachers and students.

MATHEMATICS EDUCATION IN NEW ZEALAND: HAVE WE FAILED OUR MĀORI STUDENTS?

Megan Clark
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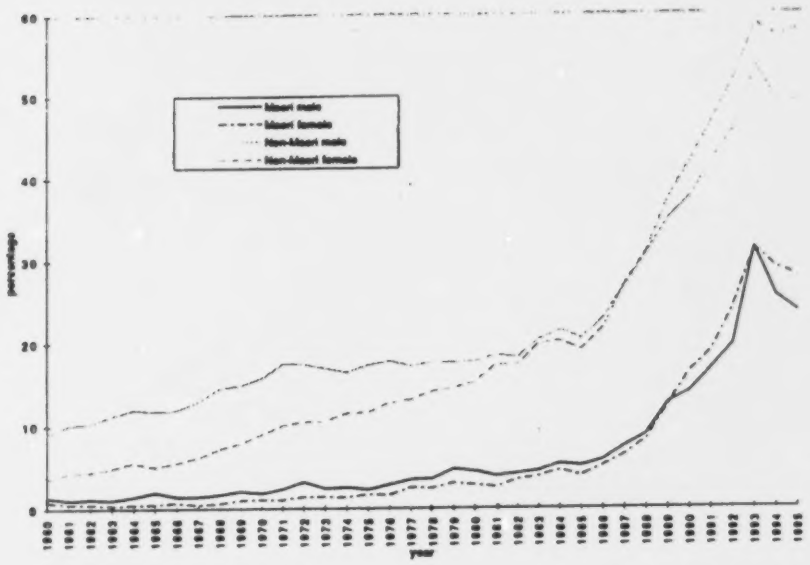
Until the middle 1980s New Zealand secondary schools had low levels of participation after the years of compulsory education, unlike many other countries. Since the mid-eighties retention rates in the senior secondary school have increased substantially as shown in Figure 1 and Table 1 (Forbes, Clark, Blithe, & Chamberlain, 1996; Pole, 1998).

In 1993 the school leaving age was raised from 15 to 16 fuelling a further increase. The school population however is not homogeneous in this regard. Māori retention rates to year 13 (form 7) from year 9 (form 3) were 3 percent in 1981 and 18 percent in 1991. In general terms the Māori retention rate is about half that of non-Māori although it shows a similar dramatic increase over the time period. Since the late 1980s the retention rate of females has been higher than that of males and consequently, female students now outnumber male students in the final year of schooling (by between 300–1200).

Figure 1

Retention Rates of Secondary Students

(% of Form 3 [Year 9] Cohort Completing Form 7 [Year 13])[1]



Mathematics has virtually full participation in year 11 (form 5, the last compulsory year of schooling for most students). In year 12 (form 6) the participation of male students in mathematics hovers between 80 percent and 90 percent but appears to be declining slightly since the mid-eighties. For female students participation in mathematics at this level is more or less constant at about 75 percent. In both years 11 and 12 the last decade has seen substantial rises in female participation in mathematics up to the current levels (*see* Table 1).

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[1] Forbes *et al.*, 1996.

Table 1

Retention of Year 9 (First Year Secondary) Entrants to Year 13 (Final Year)

1983	1993	1995
%	%	%
17	44	~50

In 1986 a marked change in final year secondary mathematics occurred. This was the replacement of the two previous courses in mathematics available at that level: "pure mathematics" and "applied mathematics" by two new ones: "mathematics with calculus" and"mathematics with statistics". This change dropped mechanics as an option for year 13 mathematics study and emphasised the importance of statistics. In year 13, while the gap in participation rates between males and females has been slightly decreased in the last decade, this improvement is overshadowed by the decline in the percentages of both males and females choosing to study mathematics which is clearly visible in Table 2.

Table 2

Percentage of Male and Female Year 13 Students Participating in each Mathematics Course in Year 13

Pure maths		Applied maths		Maths with calculus				Maths with statistics			
1985		1985		1986		1994		1986		1994	
males	females	males	females	males	females	males	females	males	females	males	females
%	%	%	%	%	%	%	%	%	%	%	%
80	60	62	38	72	49	49	29	60	40	54	42

By the early-nineties a dramatic drop was occurring in mathematics participation at this level which was more marked in male students (80 percent male students took at least one mathematics subject in year 13 (form 7) in 1992, compared with 67 percent in 1995) but still marked with female students (65 percent at least one maths subject in 1992 compared with 57 percent in 1995).

In particular, the group doing no mathematics has increased in size. It is of concern that this trend and the poor Māori retention rate to year 13 might continue and lead to a situation where women and Māori are excluded from a number of occupations because of their lack of mathematical expertise.

However mathematics with statistics and mathematics with calculus still remain (apart from English) the most popular subjects in terms of numbers studying them at year 13 (positions 1 and 3 respectively for males and 2 and 4 for females, *see* Table 3).

Overall the picture is of high numbers taking at least one mathematics paper at year 13 (peaking in 1992) but with a steady decline in the *proportion* of students at that level taking mathematics. Part of this is due to an increasing diversity of subjects available at that level which is itself due to the successful efforts to increase participation in senior secondary schooling to a much wider cross section of the community than in the past.

Of particular interest for the universities is the number and proportion of year 13 students taking *both* available mathematics courses as these have been the traditional pool from which university mathematics students are drawn. This is shown in Table 4. In the 1994 university bursary examinations, 35 percent of the students taking mathematics with calculus and 42 percent of those taking mathematics with statistics were female and these figures closely approximate the percentage of females in mathematics and statistics courses at university (Forbes *et al.*, 1996).

Table 3

The Eight Most Popular Subjects for University Bursary Examination

Male				Female			
1990	1991	1994	1995	1990	1991	1994	1995
MS	MS	MS	MS	Eng	Eng	Eng	Eng
Eng	Eng	Eng	Eng	MS	MS	MS	MS
MC	MC	MC	MC	Bio	Bio	Bio	Bio
Physics	Physics	Physics	Physics	Geog	Geog	MC	MC
Econ	Econ	Bio	Bio	MC	MC	Geog	Geog
Geog	Geog	Chem	Chem	Hist	Hist	Hist	Chem
Chem	Bio	Geog	Geog	Econ	Econ	Chem	Econ
Bio	Chem	Econ	Econ	Chem	Chem	Econ	Classics

Key: MS = mathematics with statistics
MC = mathematics with calculus

(In 1986 the total number of students offering university bursary mathematics with calculus was 5517, in 1994 it was 9211 and in 1995, 8772).

After secondary school about 23 percent of school leavers continue with formal tertiary education at a university. There has been a large increase in the total number of students enrolled in tertiary education over the last decade reflecting the expectation that students might continue with education after secondary school. Over this period there has also been a big expansion of the numbers of university students in first year so-called "service" courses such as statistics for biological and social sciences. The students in general in these courses have not taken year 13 mathematics: they may only have year 12 (or less) mathematics and are predominantly female (2:1). There has also been increased demand for entry level courses in pure mathematics for students who have only year 12 secondary mathematics. This has required some adaptation on behalf of university staff who have been used to assuming much of this material is known to students.

Table 4

Year 13 Examination Candidates Taking BOTH Maths Papers[2]

	1986	1990	1995
	%	%	%
Males	51	41	31
Females	26	22	18

[2] Clark, 1989; Forbes *et al.*, 1996.

Teachers and Schools

Inspirational and knowledgeable teaching is a major factor in retaining young people in a discipline.

The physical sciences and mathematics in New Zealand secondary schools are characterised by a lack of appropriately qualified teachers. This shortage is chronic: it is not caused by the recent expansion of the financial sector (Table 5). However, there has been a steady improvement over the last 10 years. But as the number of graduates in mathematics and applications for teaching are dropping we should be feeling concerned about this. We don't want to lose our recent gains. At Victoria University in 1995 and 1996, out of a total of 245 science graduates, five went to train as primary teachers and two as secondary teachers. A great chance was missed in the recently published *Third International Mathematics and Science Studies* in that, unlike the previous study undertaken in 1981, teachers were not asked for their mathematical or science qualifications but only for general degree and diploma status. There were indications of a continued improvement: one-eighth of mathematics teachers also taught science, compared with one-quarter in 1981, which may indicate that fewer science majors are having to fill in for mathematics shortages.

Table 5

Qualifications of Mathematics Teachers[3]

Level of qualifications	1956 %	1977 %	1979* %	1981 %	1987 %	1989 %
No. university mathematics	28	20	12–25	16**	21**	10
1st year university mathematics	18	24	18–31	–	–	23
2nd year or higher	54	56	57–69	–	–	67

* Range due to teachers who gave no information.

** Of third form (year 9) teachers.

There is a pressing need to get accurate knowledge of the qualifications of today's teachers and to continue to monitor this situation along with participation rates.

Obviously we should be upskilling and encouraging the teachers that we do have. Yet are we going about it the right way? The Science and Technology Teacher Fellowships for 1994 and 1995 were distributed roughly as in Table 6 (Royal Society, 1997) (my classification). The aims of these awards are to enable teachers to further understand the role of science and technology in New Zealand's economy and society and to promote a culture which values science and technology. Teachers of science, mathematics, and technology are eligible. The lack of awards in mathematics is very disappointing.

[3] Bull, 1956; Clark, 1981, (IEA), 1987 (PPTA survey); Forbes *et al.*, 1990; Werry, 1977.

Table 6

Science and Technology Teaching Fellowships

	1994	1995
Total awarded in:	17	19
	%	%
Biology	47	53
Physics	18	10
Chemistry and food technology	18	16
Mathematics	–	–
Earth sciences	–	10
Computer science	–	5
Equipment	5	–
Other	12	5

Teaching still depends on high numbers of females; a major factor in the adequacy or supply of mathematics teachers is still the extent to which females continue to study these subjects to year 13 and higher.

Performance

Many of you will have read reports of the results from the recent international mathematics testing that went on at years 4, 5, 8, and 9 (standards 2 and 3, and forms 2 and 3). The reports in New Zealand contain a great deal that we should be thinking about. Let me summarise some of the key results for you (Garden, 1996, 1997).

The data were collected in 1994/95. Forty-one countries took part at the year 8 and 9 level (form 2 and 3), 26 at the year 4 and 5 level (standard 3 and 4). The overall performance was just below the average of the participating countries at years 4 and 5, and at the average for years 8 and 9.

In year 9, our results were similar to those of Ireland, Sweden, Germany, England, Norway, Denmark, the United States, and Scotland (*see* Table 7).

Six percent of New Zealand students fell in the top 10 percent and 20 percent in the top one-quarter. In broad terms, we are achieving overall about the average of the group of participating countries and similarly to how we achieved in the previous study in 1981.

However within these results there is a rather unfortunate tendency to aggregation that disguises strengths and weakness. There are good things in the report: in 1981 there was a gender difference in favour of boys which has disappeared thanks to the whole community getting behind the efforts at the official level, as well as the efforts of teachers, to change this. Now there are no significant gender differences in maths performance (unlike science). New Zealand is the only country where girls do better in geometry. And New Zealand students are very good at statistics. We are poor in algebra and measurement at this level: this where we need to put in some effort. Specifically, the areas in which our students were worse than average are algebra and measurement. We are better than average in statistics and average in fractions, number, and geometry.

Table 7

Mathematics Performance on TIMSS International Scale

Year 5 equivalent	Year 4 equivalent	Year 9 equivalent	Year 8 equivalent
Singapore	Korea		
Korea	Singapore	18 countries	20 countries
Japan	Japan		
Hong Kong	Hong Kong	above	above
Netherlands	Czech Rep.		
Czech Rep.	Netherlands		
Austria	Slovenia	4 countries	1 country
Slovenia	Austria		→ *New Zealand*
Ireland	Australia	→ *New Zealand*	
Hungary	United States		
Australia	Hungary	7 countries	8 countries
United States	Ireland		
Canada	Canada		
Israel	Latvia (LSS)		
Latvia (LSS)	Scotland	11 below	9 below
Scotland	England		
England	Thailand		
Cyprus	→ *New Zealand*		
Norway	Cyprus		
→ *New Zealand*	Greece		
Greece	Portugal		
Thailand	Norway		
Portugal	Iceland		
Iceland	Iran		
Iran			
Kuwait			

As the test is primarily multiple choice and short answer it has little to say about our problem-solving abilities or performance on open-ended questions.

There are suggestions in the report (Garden, 1996) that we should consider using more whole-class teaching, a suggestion already put to the Ministry of Education by a group of maths educators in 1996 (Begg, Clark, Holton, & Neyland, 1966).

Also worth thinking about is the finding of Stigler and Hiebert in their 1997 TIMSS Video study of classrooms in the United States, Germany, and Japan. They found that 62 percent of the Japanese lessons videotaped contained work involving deductive reasoning, as did 21 percent of the German lessons, and 0 percent of the United States. We can guess where we would be. Deductive reasoning is a difficult but essential skill in mathematics—difficult to teach as well, but we shouldn't be shirking the task.

However, schools in the middle and high socio-economic brackets have nothing to worry about. As in many other countries, mathematics achievement in New Zealand is far from uniform. The majority (80 percent) of New Zealand's children have an average (across countries) achievement. Those children are right up with our traditional neighbours. There are detailed results yet to come. We are not top and never will be while we continue with a broad curriculum that has a fairly strong emphasis on the sort of complex tasks that are not tested by international studies.

The reports on the TIMSS studies (Garden, 1996, 1997) tell us something about the teachers of mathematics and certain difficulties they face in their jobs. I have mentioned the longstanding shortage of well qualified mathematics teachers. The problem is seen at its worst in the rural areas. In an attempt to solve this problem the flood of overseas teachers brings new problems. A secondary school north of Auckland with an 80 percent Māori roll has an entire mathematics department of Indian migrants, some with accents their students find very difficult to understand. In Auckland and the rural areas, no relievers are available to allow teachers to undergo further training. The mathematics advisors are stretched very thinly, with each responsible for 100–200 schools.

Singapore requires post-graduate university training for both primary and secondary teachers. Here, it is only required for secondary teachers, although increasingly many primary teachers do get a bachelor of education these days. There is a lower amount of curriculum support material for teachers in New Zealand than in most countries. Disruptive and uninterested students are rated as major problem by New Zealand teachers which may be related to the fact that we also scored highly on reported bullying.

There are, as always, myths abounding in the popular press about how to improve scores in these international tests. Myths such as: more time on maths would improve scores, as would smaller class sizes; and Asian students are the best group of students at maths. In fact, in 1981 the 3 countries with the highest achievement in year 8 (form 2), Japan, Netherlands, and Hungary, spent 101, 111, and 96 hours on mathematics per year compared with the United States and Nigeria which spent 144 and 158 respectively (second to bottom). Japan, a high achieving country, has class sizes of roughly 41 in year 8, while the United States has class sizes averaging 26. In New Zealand, Pakeha and Asian students alternate in being top, depending on the topic.

Our weaknesses are exactly the weakness pinpointed both in 1981 (in the second international study) and in 1989 (in a study I was involved in)—in measurement and number (Forbes *et al.*, 1990). These are areas which are a major part of numeracy: the part of mathematics which is not an artifact of school alone but a joint endeavour between school and life, that is, between school and home. It is in the application of mathematics in normal living that our students are weakest. Partly this may be because children now do much less mathematics at home than in the past: less baking, bathing (compared to showering), dish-washing, and sewing. We can't change our daily environment greatly but children do need to see adults doing mathematics in a relaxed and confident manner that is shared with them.

What teachers can do is provide lots of practical experience, an expectation of success, and not make too big a deal of setbacks. Mathematics is important and satisfying, but so are many other things.

People successful at mathematics often speak of experiencing homes and classrooms in which it was believed that *everyone* can do maths. There is a contrast for example, between Russia, where this is the norm and the United Kingdom where it is believed that really only a few can do mathematics successfully.

Another contributing factor to success in the countries with the top scores is the uniformly high value that those societies put on education in general and mathematics and science in particular, as well as the respect in which the teaching profession is held and the high level of subject qualification required for teaching mathematics. The value New Zealand places on teaching as a profession is very low. Japanese secondary teachers of mathematics are highly respected; they teach

one period then have a period for preparation and the end of each day have a period set aside for marking! Canada, which slightly outperforms New Zealand, is a country in which teaching is a sought after and respected profession. Furthermore, New Zealand seems unable to resolve the ambivalence that it displays towards mathematics and science. How many people would say to their children "It's really important to do well in mathematics at school" and two minutes later "I was never any good at mathematics myself"? What a mixed message. There is no way that the Ministry of Education or the Government can solve this problem for us; it is a community problem which society must face up to if it is ever to improve.

After all only about 50 percent age cohort in New Zealand go on to form 7. This is nothing like Japan or United States where over 90 percent have schooling to this level. Germany has just raised school leaving age to 20!

The reports remind us of some longstanding problems and indicates some successes (Garden 1996, 1997). We should take notice of it but take care not to apply it beyond its limitations.

To improve our performance in more than a minor way would involve a radical restructuring of our education system which we might not find acceptable. Most of the countries with high scores in this test do not have the comprehensive system of schooling favoured by New Zealand and many of the other countries in the middle band such as England, Scotland, and the United States. Most of the "top" countries discriminate by offering different curricula to particular groups of students. In New Zealand we ceased this practice some decades ago and moved instead to offering all mathematics to all students at this level. Prior to that, many students were shunted into streams which offered them only "core maths". If that is what you were offered in the third form (year 9) then there was no possibility of taking School Certificate mathematics and many career options were closed off especially as mathematics, then as now, was not only a subject in its own right but used as a gatekeeper for many professions. When I began secondary school, in a large inner city girls' college there were 8 full third forms (year 9), each averaging between 30 and 36 students. Only three of these classes were offered full mathematics.

We normally (*see* Binns, Carpenter, Elliffe, Irving, & McBride, 1987) do *very* well in year 13 (form 7) results when we do focus on a small group of students. Surely it is the "endpoint" of the system that matters not how we get there. The "outcome" (should this "outputs"?) of our current system seems excellent if the latest snippet from the IEA is anything to go by (Garden, 1998).

In 1995, nearly 2000 New Zealand students in their last year of school took part in tests to see how well they had received the knowledge and skills they needed to cope beyond school and to see how prepared they are to apply their knowledge. For example, students were asked to see how well they could interpret information in graphs.

Our senior school students are well above average (only Sweden is consistently better) in performance in this mathematical literacy study of students from 22 countries in their last year of school "regardless of whether they were still studying mathematics". This is surely an ideal result. Yet it failed to attract the media attention that our below average performance at year 5 attracted!

The high performance of New Zealand's year 13 mathematics students continues on into university. While numbers decrease, good performers from New Zealand universities can and do compete anywhere. I have students currently at Yale University, Pennsylvania State University, and the University of Lancaster. Another former student went to the University of Oxford and is now a professor at the University of Hertfordshire, another has a chair at the University of Surrey,

another is a professor at the Massachusetts Institute of Technology, and yet another at Cornell University. New Zealand is one of the few countries which has produced a mathematician who has won the Fields Medal—the mathematical equivalent of the Nobel Prize. This was won recently by Vaughan Jones, ex-Gisborne Boys High School and Auckland University for his work on knots and braids.

All of these outstanding individuals I mentioned went to state (or integrated) schools, save one. The training we offer is excellent and when that is coupled with hard work and a willingness to defer rewards, world-class mathematicians can and do emerge.

Some time ago I was told by a colleague that "women, Pacific Islanders, and Māori are not really interested in mathematics". This was put forward as an explanation for low participation and performance. They are the problem, in other words. But it is our "one stop shopping" mentality when it comes to teaching that is the problem. We know how to teach mathematics in a way that works well with a particular small group of mainly European males and Asians. So we teach everyone like that, and if it doesn't pan out, let's blame the victims!

This brings me to my area of greatest concern in teaching and in pre-service education.

Our Failure to Deliver Mathematics Education to Māori

In the 1980s women were recognised as having an inequitable chance in education and efforts went into increasing the pool of trained women which have been quite successful at the school level. We need to put equal effort into the education of Māori who are effectively locked out of career after career. Let's revisit the basic facts. Thirty-nine percent Māori leave before year 12 (form 6) compared to 15 percent of non-Māori. Thirty-five percent of young Māori leave school without formal qualifications—twice the rate of non-Māori (*see* Table 8). Whilst Māori retention to year 13 is "much increased" from the past the gap between Māori and non-Māori retention is greater than ever (*see* Figure 1). The retention rate for Māori students to year 13 is about half that of non-Māori students.

Table 8

The Gap Between Māori and Non-Māori[4]

	Percentage	
	Leaving before Form 6 [Year 12]	Leaving with no qualifications
Māori	39	35.0
Non-Māori	15	17.5

Māori participation in mathematics with calculus is very low (approximately five percent) and in mathematics with statistics it is approximately 6 percent of the candidates in each year (Burgess, 1994; Forbes *et al.*, 1996). Big improvements have occurred—Māori now constitute 11.3 percent of tertiary students which is a five-fold increase since the mid-eighties—the biggest block of these is however in the polytechnics. Of 1994 school-leavers, about eight percent of the Māori school-

[4] Te Puni Kōkiri, 1997.

leavers were destined for university, compared with about 23 percent of the non-Māori (Te Puni Kōkiri, 1993). Although Māori make up 13 percent of the New Zealand population, 20 percent of those under 15 are Māori and the Māori population will increase to about 15 percent of the New Zealand population by the year 2031 (Statistics New Zealand, 1994).

The need to improve our delivery of education to Māori is imperative. In 1996, 20 percent of the total income of Māori households came from social welfare benefits (excluding New Zealand superannuation) compared to 5 percent of the income to non-Māori households (Te Puni Kōkiri, 1997a).

Te Puni Kōkiri remains convinced that "the Ministry (of Education) has not prioritised research projects into Māori educational participation and achievement in the areas of greatest disparity" (Te Puni Kōkiri, 1997b).

We are not doing a decent or fair job of educating Māori and specifically not enough in mathematics: that, for me, is the biggest message from TIMSS. Table 9 (Ministry of Education, 1997) shows what occurs if we disaggregate the scores of Māori and non-Māori. The mean mathematics score for Māori in TIMSS is significantly lower than the New Zealand mean score. On average, Māori students performed at a significantly lower level than students in most other countries. Yet Māori and Pakeha children enter primary school with similar skills in mathematics (Yeoman, 1987). A major change in the last decade has been the introduction of bilingual teaching or immersion teaching in Māori in some classes. In 1985 the first Kura Kaupapa (Māori-language only school) was set up and by 1997 there were 54 such schools with about 3700 secondary students receiving their schooling in mathematics in Māori. There is much evidence that "mother tongue" teaching provides for more equality of opportunity. It is likely to ease the young child's transition from home to school and foster self-esteem, general linguistic development, and conceptual growth.

While the TIMSS report highlights our inability to close the achievement gap between Māori and non-Māori it does contain indications that Māori students in Kura Kaupapa immersion schools are doing better than other Māori. Recent local findings (Aspin, 1995) are that 10- and 11-year-olds who had received all instruction in Māori at a Kura Kaupapa scored higher on a mathematics test than students receiving instruction only in English. The Kura Kaupapa students did better in both Māori and English versions of the test and best of all in the English version. This result is confounded by socio-economic status and being a highly selected group but it's our first sign of success and needs much more resourcing. Teachers who teach mathematics in immersion schools not only have to teach it but, by and large, have to provide all their own materials. We have only a very small band of people able to do this, all of whom must be close to burn out. They need tangible support. If we can get Māori performing in mathematics at the same level as everyone else, then we have really nothing to worry about.

The signs are that Kura Kaupapa and the bilingual units are managing this task. But only 15 percent of young Māori are educated in this way. We are failing 85 percent. They should be the primary focus of mathematics education for the next decade, until we achieve equity of opportunity and expectation.

Expectations of Māori students are far too low. Too many Māori are still being held back at 6 years of age. There is much evidence that this is counter-productive and when children of equivalent ability and achievement are compared, those kept back stay behind while those advanced catch up. We often make the mistake of assuming that if someone is having difficulty with mathematics at

a certain age that that difficulty will persist. This is not so: many boys, in particular, who struggling at 7 and 8 years of age can (if they are encouraged and expected to) perform at a much higher level a few years down the track. Here are one such child's scores from the *Progressive Achievement Tests, Mathematics* at standards 2, 3 and 4 (years 4, 5, and 6): 65, 76, 85. Many teachers regard the first three years at school as a key time and the transition to school as providing children with more opportunities to learn to become independent (Higgins, 1998). So while students who are "kept back" are suffering all the other indignities of this treatment, they are also "kept back" from independence and, in particular, from independent learning. The dangers of this seem self-evident. "Independent time gives them more confidence and teaches them to learn to do things on their own" (Higgins, 1998). Furthermore teachers of year 2 children increase their emphasis on "work" (Wylie & Smith, 1993). Thus students who are "kept back", are kept back from work as well.

Higher up the school too many Māori students are not encouraged to participate in national examinations: 87 percent of Māori in their third year of secondary school (year 11) sat School Certificate in 1990, compared with 94 percent non-Māori. Seventy-seven percent sat Sixth Form Certificate (year 12), compared to 86 percent non-Māori (Davies & Nicoll, 1993). In the senior secondary school much of the gap in performance and participation of Māori compared to non-Māori can be attributed to Māori students' lesser participation and performance in the School Certificate examination. The Ministry of Education (1992) acknowledges that timed written tasks may not be reliable indicators of Māori performance but the School Certificate examination is still largely a collection of such tasks (Clark, 1996). Marked differences between Māori and non-Māori students in mathematics attained in year 11 have remained virtually unchanged for the last 20 years (Forbes *et al.*, 1990). In 1995 the mean School Certificate mathematics score for Māori was 43 percent compared with 55.5 percent for non-Māori with the differential showing no signs of lessening. In Sixth Form Certificate Mathematics the most striking feature is the differential in attainment for non-Maori and Māori. Approximately 30 percent non-Māori females (and 25 percent for non-Māori males) were awarded one of the top three grades from 1992 to 1995. Over this period the percentage of Māori males awarded these grades dropped from 13 percent to eight percent with Māori females roughly constant at 11 percent.

In university bursary mathematics too, the differential remains. On average, the mean mark awarded to Māori is eight to nine percent lower than that awarded to non-Māori in both papers (Forbes *et al.*, 1996).

It is our task now to "raise our expectations" of Māori students. We have done it for women; let's do it again.

We know it is possible. Looking at the second and third international mathematics studies, from 1981 to 1994 the percentage of Māori whose achievement was in the top 20 percent (nearly doubled from five percent to nine percent). (But 35 percent and 33 percent Māori were in the bottom 20 percent of scores.) (Ministry of Education, 1997).

A major function of education is the transmission or construction of culture from generation to generation. Culture is a system for creating, sending and storing information. Students learn from education a particular cultural complex called "mathematics" and the person who becomes expert in mathematics has their thinking about the nature of the world shaped by their study. This cultural function helps to explain the way in which Māori and Pacific Islands students are often not expected to be good at mathematics. For over a century now the Western countries have seen mathematics

as "theirs" belonging to their (Greco-Roman) tradition and conveniently ignoring the contribution of Hindu, Chinese, and Arabic traditions. Many of us need to work through some issues here. The big question for educators is "how" we pass on the culture to students—whether in doing so we enhance or damage their existing culture.

> It is pre-eminently the task of education so to induct people into culture that their personal freedom and creativity are preserved and indeed enhanced... The object of education is to make culture more of a resource and less of a determinant.
>
> (Stenhouse, 1967)

We need mathematics education that serves the needs of Māori students.

Table 9

Māori and Non-Māori Mathematics Performance on TIMSS International Scale

Year 5 equivalent	Year 4 equivalent	Year 9 equivalent	Year 8 equivalent
Singapore	Korea		
Korea	Singapore		
Japan	Japan		
Hong Kong	Hong Kong	18 countries	20 countries
Netherlands	Czech Rep.		
Czech Rep.	Netherlands	above	above
Austria	Slovenia		
Slovenia	Austria		
Ireland	Australia	4 countries	Germany
Hungary	United States	➡ *NZ non-Māori*	➡ *NZ non-Māori*
Australia	Hungary	Germany	
United States	Ireland	➡ *New Zealand*	3 countries
Canada	Canada		
Israel	Latvia (LSS)		➡ *New Zealand*
Latvia (LSS)	Scotland		
Scotland	England	6 countries	5 countries
England	➡ *NZ non-Māori*		
➡ *NZ non-Māori*	Thailand		
Cyprus	➡ *New Zealand*		
Norway	Cyprus	6 below	4 below
➡ *New Zealand*	Greece		
Greece	Portugal	➡ *NZ Māori*	➡ *NZ Māori*
Thailand	Norway		
Portugal	Iceland	5 countries	5 countries
Iceland	➡ *NZ Māori*		
➡ *NZ Māori*	Iran		
Iran			
Kuwait			

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A CONCEPTUAL DEVELOPMENT FOR MULTIPLICATION: THINKING STRATEGIES IN PLACE OF TABLE MEMORISATION

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Understanding the multiplication concept is an important aspect in the development of multiplication. It is the key to problem solving involving both multiplication and division, and provides the basis for most higher order mathematics, ranging from algebra to ratio. It is also crucial to the meaningful building up of facility with the basic facts on which computation relies. This paper will present practical activities for developing the full range of multiplication concepts and constructing an integrated base of multiplication facts in an effective and efficient manner.

The multiplication concept grows out of experiences with repeated addition but there is a need to establish it as a notion in its own right distinct from addition. Indeed, informal grouping activities may suggest that 2 fours is different from 4 twos, although in each representation the total amount, 8, is the same.

2 fours 4 twos

Making groups also encourages counting to find the total, yet repeatedly adding is not only impractical when larger numbers are involved, it also hinders the application of multiplication to problem situations. Not that grouping should be avoided, nor a repeated addition interpretation discouraged. In the early stages, this method is the only means children will have to find the total amount. However, a prolonged application of it may promote inefficient strategies for later work with multiplication facts:

9 fours? 4 and 4 is 8 and 4 is 12 and 4 is .....

On the other hand, placing objects in an array enables both interpretations to be seen and makes it obvious that the two products must be the same:

• • • •	4 in each row	4	• •	2 in each row	2
• • • •	2 rows	× 2	• •	4 rows	× 4
	2 fours are 8	8	• •	4 twos are 8	8

An awareness of what is shown, 2 rows each with 4 objects or 4 rows with 2 objects, then leads to the recording and a reading from bottom to top consistent with the algorithm for larger numbers.

The use of arrays also helps children to discriminate between multiplication and addition, and lays the groundwork for mastering the basic facts by providing a link to earlier numeration and addition understanding. Such an approach, which provides thinking strategies to obtain multiplication facts before introducing practice to obtain automatic recall, enables them to be developed in a meaningful way as well as making them easier for children to learn. In contrast, when thinking strategies are not taught, many children use methods based on counting, repeated addition, or rote learning. This has even been encouraged by the tradition of learning multiplication facts by the mindless repetition of "tables", as if the learners were devoid of any ability to think for themselves.

Not only was such an approach a complete denial of any earlier learning in numeration or addition, which in reality provide a very good basis for constructing knowledge of the multiplication facts, but it was also terribly inefficient. Never was it mentioned that each fact was repeated twice in reciting the tables, nor was the link between these two forms brought out as a basis for learning. In contrast, an approach using a visual array model makes very explicit use of earlier learning, and attitudes to learning, for the power of thinking strategies is something that strikes children and appeals to their wish to understand and have control over their own mathematical thinking.

Many multiplication facts are already known as a consequence of mastery of the addition facts and earlier numeration knowledge: they merely need to be reinterpreted and extended by means of the array model. For example, the "twos facts" are really just a restatement of the addition doubles:

6	• • • • • •	6
× 2	• • • • • •	+6
	2 sixes are	12

Other facts can then be built onto this knowledge, using the array as a visual model to show what is to be done:

6	• • • • • •		
× 3	• • • • • •	2 sixes are	12
	• • • • • •	and	6
	3 sixes are		18

The facts can then be introduced in an order that allows one set of facts to be constructed readily from those already known, using the array model to guide the thinking that is required.

A Teaching Sequence for Introducing the Multiplication Facts

1. Build up an understanding of the multiplication concept in terms of arrays.

2. Develop thinking strategies that allow answers for all of the facts that require a particular way of thinking to be found before practice (including games) is begun. In this development, note that:
   - Materials and associated activities are best used to introduce the thinking strategy.
   - Games and worksheets can be used to consolidate the particular way of thinking.
   - Automaticity should be built up for each cluster of facts before those facts associated with the next way of thinking are built up.
   - Care needs to be taken that the learner can discriminate the way of thinking to generate a given cluster from that used for the previous cluster.

3. An order for developing the clusters of facts, based on the use of arrays, which leads to ready mastery is:

twos:	as shown on page 21, these are really a restatement of the addition doubles.
fours:	these are most readily built up using 2 of the twos as shown in Figure 2.
threes:	these are most readily built up using the corresponding twos fact and 1 more row, as shown in Figure 2.
fives:	can be related to the reading of minutes on a clock.
sixes:	these are most readily built up using the corresponding fives fact and 1 more row, as shown in Figure 2.
nines:	examining the nines facts known from the facts already met (2 nines, 4 nines, 3 nines, 5 nines, and 6 nines) reveals a pattern that the digits always sum to 0. For example, 7 nines must be close to 7 tens, so it will be in the sixties. Since 6 and 3 are 9, 7 nines are 63.
8 eights:	is readily seen as 4 eights and 4 eights, as shown in Figure 3.
7 sevens:	can be seen within the array as 5 sevens and 2 sevens, *or* 3 sevens and 4 sevens, *or* 6 sevens and seven, as shown in Figure 3.
8 sevens:	can readily be seen as 7 sevens (49) and 1 more seven or 56, thus 7 eights as well.

When the concept and sufficient of the multiplication facts are known to provide a firm basis for multiplication, the *zeros* and *ones* facts can be developed through a similar use of the array concept.

It is helpful to trace this developmental sequence by shading the appropriate row and column a different colour for each cluster of facts, in order to see how the master of one cluster not only provides the basis for those that follow, but also that the number of new facts to be learned reduces for each new cluster.

Figure 1

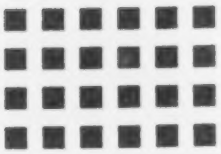
Tracing the developmental sequence for the multiplication facts

x	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

Figure 2

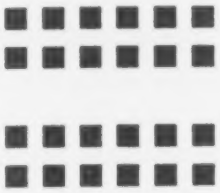
Using Arrays to Master Multiplication Facts: Fours, Threes, and Sixes

Make this array

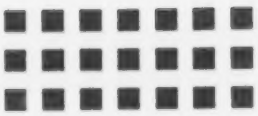


4 sixes

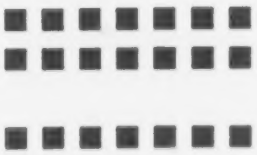
Then move the rows – what facts are shown now?



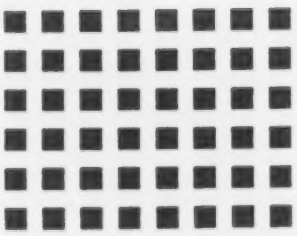
2 sixes and 2 sixes



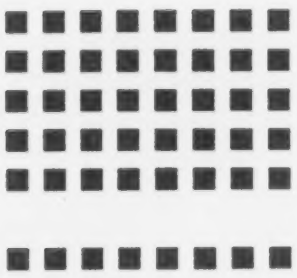
3 sevens



2 sevens and 1 seven



6 sixes

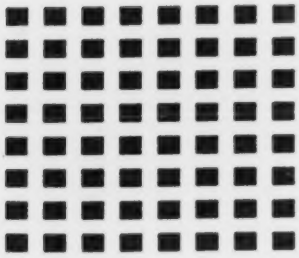


6 sixes and 1 six

Figure 3

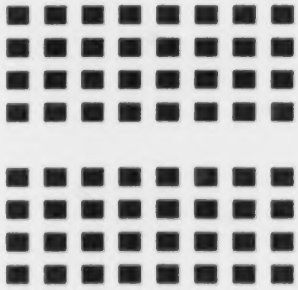
Using Arrays to Master the Multipication Facts: 8 Eights, 7 Sevens, and 8 Sevens

Make this array

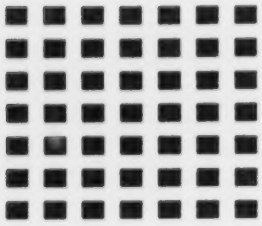


8 eights

Then move the rows – what facts are shown now?

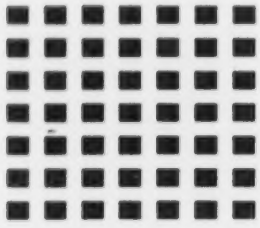


4 eights and 4 eights



7 sevens

or

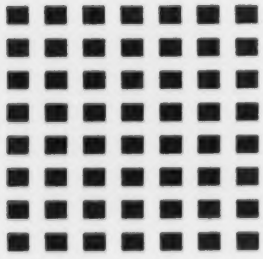


3 sevens and 4 sevens

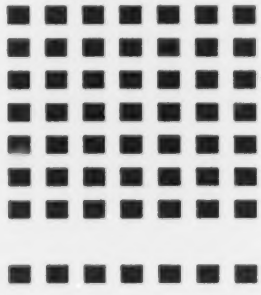
2 sevens and 5 sevens

or

6 sevens and 1 seven



8 sevens



7 sevens and 1 seven

The thinking for multiplication at all levels is more abstract than that needed for the other operations. Just as the multiplication concept built on the known addition concept rather than the story situations in which multiplication could arise, the strategies for mastering multiplication facts build on the use of arrays to allow a visualisation of the thinking that is needed. Materials used to develop these ways of thinking provide a link to earlier knowledge, while the simple language used for multiplication helps to ensure a smooth build up of known facts.

The development of thinking strategies associated with meaningful basic-fact learning also contributes to the thinking required for effective problem solving. Solutions to many problems are assisted by an ability to interpret what is needed in terms of an array or area model. Such problems require the translation of familiar knowledge, building it into novel procedures in a similar manner to the way in which the multiplication facts build on earlier numeration understanding or addition fact knowledge. Thus, rather than being a rather tedious aspect of initial learning in multiplication, building up automaticity is an important contributor to the development of mathematical thinking.

It is also the case that of all aspects of elementary mathematics that give children a sense of security about their own abilities, "multiplication tables" feature highest. Children who do not know these facts have the feeling that they know very little about mathematics and this thinking can very quickly be self-fulfilling given the crucial part confidence plays in learning and applying a subject like mathematics. Even worse, many children who do not know some of the multiplication facts generalise to believe that they do not know multiplication at all and even that they "are no good at mathematics". Teaching which allows children to construct their own knowledge of the multiplication facts using arrays has proven to be readily accessible to young children and to be a key element in developing both their problem solving ability and their self-confidence.

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PROMOTING NUMBER SENSE RATHER THAN SIMPLY PROVIDING NUMBER SKILLS

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It is ironic that many people still view mathematics as facts, rules, and formulas when mathematics as a sense-making process is highly valued because of the need for a more technologically skilled work force (McIntosh, Reys, & Reys, 1992). Indeed, existing school mathematics programmes often appear to promote the acquisition of standardised procedures for manipulating numbers without adequate understanding of their meanings (Campione, Brown, & Connell, 1989; Greeno, 1991; Hiebert & Carpenter, 1992). As a consequence, children come to believe that mathematics is mainly concerned with finding answers quickly that are then only judged by a teacher or text, rather than responses that can be self-assessed in terms of the thinking processes that were used or the context in which they are set. In contrast to such a rule-based approach, number sense, like problem solving, needs to emphasise a way of thinking about numbers rather than a body of knowledge that can simply be transmitted from teacher to student (Markovits & Sowder, 1994). Indeed, teaching that encourages number sense is the kind of instruction that will promote meaningful mathematics learning across the board (Sowder, 1992).

Yet, number sense is an elusive concept. Although it is featured in most contemporary curriculum documents, it is not well defined and is more often described in terms of children and adults who do not exhibit sense in using or interpreting numbers. Laborious ways of completing calculations are often cited in contrast to more efficient or effective ways revealed by thinking about the numbers or operations involved. For instance, many small-business people find increases or decreases in price by multiplying by the percentage, dividing by 100 and then adding or subtracting the result rather than using a decimal fraction equivalent or the percentage key on their calculator. Standard procedures are often used regardless of context—students happily report that a 20-year-old would be 3m tall since his height at 10 years of age was 1.5m (Markovits, Hershkowitz, & Bruckheimer, 1989, p. 53). Other examples also reveal little inclination to reflect on the reasonableness of results:

> Sue and her father deliver milk every day of the week. Each morning, starting at 6 o'clock, they deliver to 129 homes and each afternoon, starting at 4 o'clock, they deliver to 64 homes. How many deliveries do they make in one week?
>
> ```
>   129
>    64
> ------
>  8256
> ------
>    × 7
> ------
> 57792      57 792 deliveries in a week
> ------
> ```
>
> Would 2 persons be able to make 57 792 household deliveries in 1 week?

When the "obvious" calculation is exhausted, students frequently resort to saying that there is insufficient information to solve the problem rather than consider that their interpretation of what is required may be at fault:

The city council charges each householder yearly rates. On Marcia's last bill, she was charged $38.25 per quarter for water and $41.25 per quarter for rubbish collection. How much did she pay daily for water and rubbish collection?

$$365 \div 4 = 91.25$$

Can't do problem as can't have quarter of day!!

quarter of a day vs 1 quarter of a year

Only when a result appears conflicting is the possibility of an error considered and then most often the answer is simply recalculated using the same procedure. Indeed, even though in school there would have been some attention to the idea that answers could be checked or assessed in a variety of ways, for many children and adults the only strategy is simply to "do" the calculation again (McIntosh, Reys, & Reys, 1997, p. viii).

Once it was assumed that the teaching of number automatically brought about a sense of, and control over what was happening, as this was the intention of those who organised the curriculum and implemented the teaching. The first reference to a need to expressly emphasise this aspect of mathematics was in the Cockcroft report (1982), where it was referred to as "at homeness with number". More recently, in Australia the *National Statement on Mathematics* has proposed that "all people need to develop a good sense of number, that is, ease and familiarity with, and intuition about, numbers" (AEC, 1991, p. 107). Similarly, in the United States, the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* (NCTM, 1989) emphasised the notion of number sense as a fundamental aspect of mathematics learning involving well-understood number meanings, multiple interpretations and representations, appreciation of the effect of operations together with a system of personal benchmarks. However, these ideas need to be developed more explicitly if a foundation is to be provided for teachers and learners to build sense making as a fundamental aspect of their mathematics learning.

Number sense is best characterised in terms of the ways of thinking that need to be developed along with the fundamental notions of number. In particular, it needs:

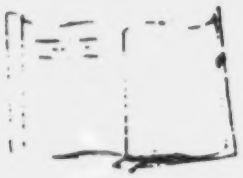
- well-understood numeration in order to think about numbers flexibly in terms of renaming and interpreting them in a variety of ways

At the Yaralunga Primary School Bush Dance there were 357 pupils and parents. Five pairs were needed to form each group of dancers for *Strip the Willow*. How many groups of dancers were they able to form?

a number expander shows that 357 has 35 tens 7 ones

- a meaningful grasp of computation, especially a full awareness of the various concepts that relate to an operation, the variety of ways in which they occur and can be represented, and a feeling for the effect of operations, including those done differently or more efficiently with a calculator.

A book is opened, and the product of the two page numbers that appear is 1190. What are the two page numbers?



the square root of 1190 shows that the numbers must be 34 and 35

- the linking of these understandings to provide an inclination, as well as an ability, to interpret processes and information used in order to communicate about and with mathematics.

As McIntosh, Reys, & Reys (1992) have pointed out, such a foundation is necessary to provide the flexibility and thinking strategies that number sense requires. In this way, personal *invented* algorithms can be created and applied to problem situations while a capacity to *reflect* on answers given by these and by learned algorithms (mental, calculator, estimation, or paper and pencil) can come to the fore. The need to make sense of the relationship between a problem context and any solutions obtained can then be seen as the essence of mathematics in contrast to the mere provision of an answer to a computation that may or may not accord with the problem being addressed.

Activities to Promote Number Sense

While many of the activities carried out in the teaching of number develop the basis of number sense, often a student is able to adopt the processes they reflect in a procedural manner and one that is focused on standard forms of questions and the provision of answers as quickly as possible. Indeed, one of the difficulties in teaching is to determine whether a student who responds in the way a teacher wishes actually knows what the teacher expects them to know or whether the student has learnt to answer in the manner that the teacher expects. One way around this dilemma is to ask different questions in a less direct way, focusing more on the understanding that underpins a response than the answer to a problem. For example, asking questions in the form "how do you know that 34 567 is less than 33 987?" rather than "which number is less, 34 567 or 33 987?" Providing a problem with its answer and asking for reasons to justify the answer not only shows that it is the thinking that is behind the response that is important, but it allows a teacher to more readily see the misconceptions or simplistic views of thinking that a child might exhibit. Some examples (Booker, in press):

- How do you know that 45 is between 46 and 44?
- How do you know that multiples of 5 must end in 5 or 0?
- How do you know that five thousand and thirteen is not written 500013?
- How do you know that one-quarter percent is not 25%?
- Why is 3.256×10^7 greater than 8.9×10^6?
- Why is 600 a good estimate for 149 + 152 + 148 + 147?
- Why is 5367 rounded to the nearest ten 5370 and not 5000?

A variety of formats for questions of this kind have been provided by Glatzer & Glatzer (1989, 1994) and the questions that are included here have been based on their frameworks. Other categories of questions that they have developed include:

- Writing ways in which statements are the same and different.
- Which number, expression or calculation does not belong?
- What if questions in which an example needs to be provided to match given conditions.

Some examples that fit these types of question are:

- State at least 3 ways in which 923 and 1189 are the same and at least 3 ways in which they are different.
- (78, 79, 82, 87)—which number does not belong? Give reasons for your choice. Can you find reasons why each of the numbers would be the one not to belong?
- What type of number results when an even number and an odd number are added?
- What type of number results when an even number and an odd number are multiplied?
- What are the different ways that that apples can be shared equally if there are 72 apples and less than 20 people?
- If you use the digits 5, 7, and 2, what is the decimal fraction with two decimal places that is closest to 6?

Of course, asking open-ended questions is another way of encouraging students to examine their own knowledge rather than simply look for something that they might have done before:

- Find two numbers which multiply to give 1 000 000 so that neither number contains any zeros.

Mathematics as Sense Making

If all new mathematical ideas are constructed with questions of this form at the heart of classroom activity, new concepts and strategies will be built up efficiently from established ways of thinking and proceeding. The learning will also be more effective as a connected view of the subject will be formed. In turn, this will lead to students who view learning as a problem-solving task and who are autonomous in their ability to use, apply, and extend mathematics. Mathematics will be viewed as a way of thinking about the world that will enable each school leaver to participate fully in all aspects of their social and working life.

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ASSESSMENT RESOURCE BANKS IN MATHEMATICS: HOW CAN TESTING HELP TEACHING AND LEARNING?

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Teachers have an ongoing need to test the ability of students to meet the specific objectives laid down in the curriculum. One tool that is available to help them in this task is the Assessment Resource Bank (ARB). This is a bank of assessment items freely available to New Zealand teachers via the Internet. Because it is computer based, it allows the user to employ various search strategies to locate suitable resources for their own particular needs. The ARB is flexible enough to allow for a range of assessment including formative and diagnostic assessment. The way in which children answer mathematical questions gives strong clues as to what thinking processes may have lead to incorrect answers. These can then be turned into diagnostic information. Often the diagnostics will point up misconceptions in strands of mathematics other than the one actually being assessed. Some examples will be shown to illustrate both how the ARB operates and how its diagnostic potential can loop back into the teaching and learning process.

Introduction

The Assessment Resource Bank (ARB) is a project funded by the Ministry of Education to provide an online database of resources that can be used in assessment. The New Zealand Council for Educational Research (NZCER) has been contracted to produce the ARB. These resources are currently available free to any school in New Zealand on application to NZCER. There are separate banks for mathematics, science, and English. The banks allow the user to select from a wide range of assessment tasks which have been developed in conjunction with classroom teachers.

Each resource has been trialled on about 200 students to make sure of the items suitability and to calibrate the level of difficulty. A classical analysis of the data was conducted, using the computer program Quest, to produce the level of difficulty of each question and its power of discrimination using the point-biserial statistic. This approach centres on analysing correct responses, especially for constructed responses. This ensures that each resource has reliability and validity.

While the ARB is closely related to more familiar item banks, it has two distinguishing features. Firstly, it uses the online search facilities available on the Internet to help the user locate questions of particular relevance to their own classroom or school. Secondly, it is designed to include not just multiple choice questions but also short and long answer questions as well as practical activities. This makes it a far more flexible tool.

While working on the mathematics ARB project it became increasingly clear that the alternative (that is, "wrong") answers that children came up with provided valuable insight into childrens' thinking and to common misconceptions that they have. The later part of this paper describes how these wrong responses are being turned into diagnostic information and shows some examples of questions with related diagnostic information. A clear understanding of the causes of misunderstanding can then loop back into teaching that targets the specific underlying problem.

The paper falls into two halves. Firstly the ARB will be described. This is followed by a discussion on diagnostics.

The Assessment Resource Bank

What Are the ARBs in Mathematics?

The ARBs are expanding collections of assessment resources developed by NZCER under contract to the Ministry of Education. They incorporate a computerised database management system available on the Internet. This allows a user friendly, easy to use system of classifying and searching for the particular resources to match curriculum and teaching objectives. The ARBs are explicitly modelled on the New Zealand curriculum statements in mathematics and science. They include a range of assessment styles ranging from multiple choice, short and longer constructed responses, as well as practical tasks. This makes them a highly-flexible tool that can complement existing assessment approaches already in use at a school.

Each individual resource contains a performance task for the student to do. This has an associated marking guide. The difficulty of the task has been calibrated on a sample of students and states how hard the task was for students at the appropriate level. For some resources, diagnostic information is given to help identify common errors and the likely misconceptions that led to the wrong answers. Every resource is classified by the strand, objective, curriculum level, achievement objective, and resource type. A list of keywords are also attached to each resource (*see* Figure 1).

Figure 1

Content of Resources in the ARB

1. **A performance task.**
2. **A marking guide,** giving a model answer and associated score.
3. **The classification of that resource** (Strand, Objective, Level, etc.)
4. **The difficulty of the resource** stating how hard the resource is for students at the given level.
5. **Diagnostic information,** to help identify areas of misunderstanding or confusion.

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Access to the ARB

This can be done by logging onto the NZCER homepage on the Internet. This is located at <http://www.nzcer.org.nz/nzcer3/reg.htm>. On the NZCER homepage, the user clicks on the ARB line and from here can peruse some background information on the system. Before entering the bank the user must have a password. To apply for this click on *Click here to receive a password,* fill in the form which appears and then click on the *Click when finished* button at the bottom of the page. To obtain a password, you need to be applying as a teacher, a school, or an approved education professional.

Once your password has been returned to you, you may enter the mathematics or science bank by clicking on the appropriate button. The banks may then be searched for appropriate resources. When a resource has been located it may be printed or copied to a file for actual use. Instructions for this are given online.

Using the ARB

Resources in the bank can be retrieved by using the search page, *see* Figure 2. This page contains a box for each of the five classification fields and the keyword field that are used to define each resource. The five classifications are strand, achievement objective, curriculum level, resource type, and process skill. Further information of each of these can again be found online by clicking on *Further information on the classification system.* It is best to conduct a broad search first (for example, Number, Level 3), and then narrow it down as you proceed. If, for example, you select Number, Objective 1, Level 3, and Communicating mathematical ideas, you may only have a small number (or no) resources to choose from.

Once a search path is defined click on the *Search* button. All resources which match your classification choices will be listed. By clicking on the *number* of any one of those listed, the resource will be shown. The first page gives the performance task. By clicking on the *scoring* button, the marking guide will be shown along with the difficulty of each part of the question and sometimes some diagnostic information.

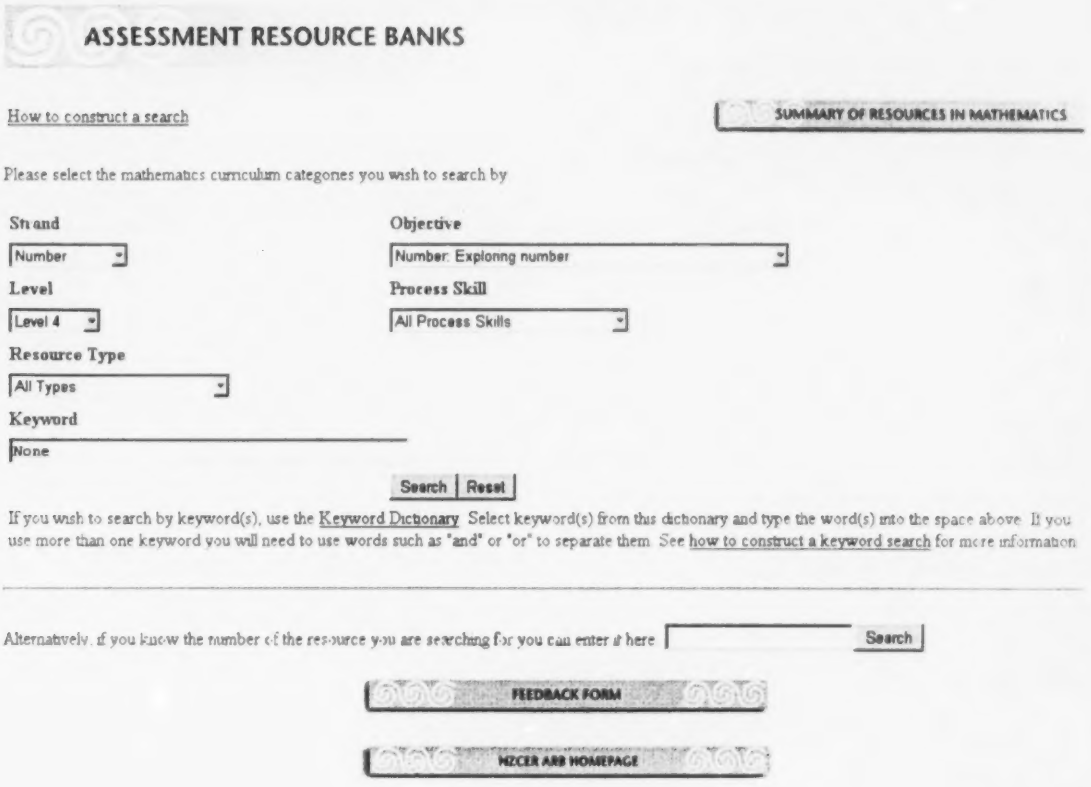
Each resource consists of a performance task. This is what the student is asked to do. It is on page one of the resource. Page two gives the marking guide, the difficulty of each part of the task (on a five-point scale from very hard to very easy), and for some resources diagnostic information.

What Can the Resources be Used For?

The ARBs provide material for six main school-based assessment purposes. These include formative, diagnostic, and summative assessment; school-wide assessment; pre- and post-tests; and examples of exemplars. Further information on these can be found online by clicking on the button *What can the resources be used for?* and then on the specific area of assessment. Clearly, a wide range of assessment is possible and the choice of what to use and in what mode of assessment is left in the hands of the professional teacher or school. The teacher can choose to amend the task, but then the calibration data will no longer be relevant.

Figure 2

Mathematics Search Page from the ARB


ASSESSMENT RESOURCE BANKS
How to construct a search
SUMMARY OF RESOURCES IN MATHEMATICS
Please select the mathematics curriculum categories you wish to search by
Strand
Number
Objective
Number: Exploring number
Level
Level 4
Process Skill
All Process Skills
Resource Type
All Types
Keyword
None
Search
Reset
If you wish to search by keyword(s), use the Keyword Dictionary. Select keyword(s) from this dictionary and type the word(s) into the space above. If you use more than one keyword you will need to use words such as "and" or "or" to separate them. See how to construct a keyword search for more information.
Alternatively, if you know the number of the resource you are searching for you can enter it here
Search
FEEDBACK FORM
NZCER ARB HOMEPAGE

Diagnostic Assessment

The area of diagnostic assessment will form the balance of this paper. This is aimed at assisting the teacher to isolate strengths and weaknesses in learning. Commonly occurring wrong answers are identified, and likely reasons for the errors are made. This can loop back into the teaching cycle by focusing on explicit learning needs of the student that the assessment indicates.

Creating Individual Diagnostics

To produce diagnostic information on a question, the pattern of incorrect answers must be analysed. This is a routine task for multiple-choice questions, but far less trivial for constructed responses. If the constructed response is numerical in nature (as it often is in mathematics) then the analysis is relatively straight forward, at least in theory. One only needs to build up a frequency distribution of all the incorrect responses an look for patterns in them. If the constructed response is of a written nature then the task is both more time consuming and more subjective. Essentially it becomes like a content analysis approach for a questionnaire. Each written response must be coded as falling into groupings of similar responses. These can then be tabulated and patterns looked for.

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In practice we do not have the resources to analyse constructed responses this way. Instead a two-pronged approach is used. Firstly our markers look for common incorrect responses as they mark. The markers can sometimes infer the misconception from clues the students' scripts (working etc.). A subsample of approximately 30–40 scripts are then subjected to a more rigorous examination for errors. This makes the marking a far more interesting, less perfunctory job. The marker must use intelligent marking, asking "What pattern of wrong answers are there?" and inferential marking asking "What might have caused these wrong answers?". Sometimes there are clues on the script that indicate the students thinking, especially if they show their working.

The two criteria for establishing diagnostics are:

- Is the error common to many students?
- Is there a clearly definable misconception that the response indicates?

For an example of this, see how the responses to the following question from ARB were analysed:

> Find the sum of the smallest and the largest of these numbers.
> 0.75, 0.2, 0.5, 0.25.

- An answer of 0.55 indicates a confusion between sum and difference. This was not found in practice.
- An answer of 1.7 indicates that all the numbers were added.
- The unlikely sounding response of 0.77 was often spotted.

The last two responses were used on the resource as diagnostic information. The first was rejected because it contravened the concept of being common. The middle one was common and had a plausible explanation. The last one, upon reflection, has a plausible explanation too. The student appears to be adding 0.02, showing some confusion as the significance of zeros and of place value.

It must be pointed out here that the cause of the error has been inferred from the answer. It would be more satisfactory to be able to ask the student what their thinking processes were. This would be an enormous task to perform on hundreds of questions each on about 200 students. The approach of inferring the misconception is ideal for numerical responses. It is highly probable that the numerical answer has been arrived at via the anticipated method with only a slight chance that an alternative logic has been applied.

The diagnostics are written onto each resource with the common errors and the likely misconception identified. Often this is accompanied by the likely calculation that the student has used. These diagnostics are specific to the actual questions in an individual resource. For classroom use the teacher may wish to have a face-to-face conference with an individual student to establish the cause of error. They may also have a small-group or a whole-class discussion or teaching session. The diagnostic information can be used in these sessions as starting points for the teacher to help them establish the true cause of error.

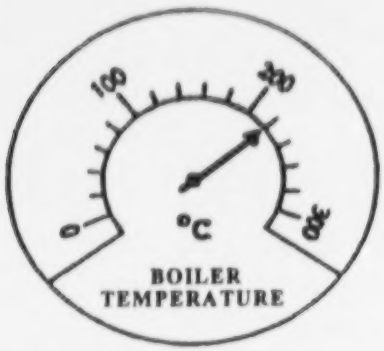
The cause of the error may not lie in the strand of the testing but in another strand of mathematics. For example, *see* Figure 3: while measurement is being tested, the diagnostic information shows that the students' misunderstandings lie in number understanding, specifically in the idea of place value.

Figure 3

Example of Where the Cause of the Error is Not in the Strand Being Tested

Students were asked to read a thermometer. Their answers show misconceptions in how to read a scale (a measurement objective) as well as misunderstandings about the place value of numbers (a number objective).

a)



The diagram shows the school boiler temperature gauge. How hot is the boiler?

Diagnostic Information

	Common Error	Likely Reason
a)	201 (200.1)	Treats divisions as 1° (or 0.1°) rather than 20°. Place value error.
	202	Treats divisions as 2° rather than 20°. Place value error.
	210	Treats divisions as 10° rather than 20°.

Types of Diagnostics

In the ARB, the model for the diagnostics reflects the approach of Newman who gives a spectrum of where the error may occur (Newman, 1997). This ranges from a reading problem to a comprehension problem. Even if these are achieved the student may have a transformation problem where they cannot turn the problem into the appropriate mathematics. The mathematics itself may be incorrectly done. Even if all this is correct, the answer may be encoded incorrectly, *see* Figure 4 for an example of a problem where each of these categories of error could occur. In the ARB, the diagnostics are written specific to the actual resource, but may fall into any of the five categories above. Other authors have concentrated their efforts on the transformation and mathematical activities themselves (see Booker, 1994; Doig, 1990; Kirsch & Mosenthal, 1993; Palmer, Kays, Smith, & Doig, 1994).

Figure 4

Different Categories of Errors

Example

If each of the 36 students in Mrs White's class brought 24 cookies for the school party, how many cookies would be available for the party?

Answer	Likely Reason	Error Category
24	Ignores the word "each"	Reading
36	Doesn't know what to do. Guesses ???	Comprehension
60	Adds numbers instead of multiplying	Transformation
216	Incorrect multiplication – Fails to displace subtotal 36 x 24 144 72 216	Computation (Algorithm)
894	Encodes the 6 as a 9	Encoding

Some errors occur in many different resources and are of a more general nature. The user can easily generalise the specific comment to the generic one. These general diagnostics on student errors are being built up to create a lists of errors commonly made by students in each of the five contextual strands. These are anticipated to be similar to the list built up by Palmer *et al.* (1994) in the number strand only. At this point, diagnostics have only been included in some resources currently available on the ARB so the lists are still in their infancy. The low number of resources with diagnostics reflects the extra time it takes to produce them, and also that the idea of including diagnostics only emerged more recently in the life of the project.

The ARB has in no way been able to identify all the possible errors and misconceptions that students have made or could make when attempting resources in the banks. The moral is that it is well worth reflecting on the processes that a student may have used to come up with an incorrect answer. Often the answer alone will provide sufficient clues. If this is followed up with one-to-one conferencing of students, then the problem can be confirmed and a strategy to remedy it can then be developed.

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